Math Review

CFA L1 Standard

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To Begin with

ChatGPT is a good coach if and only if you have a good understanding of what you are asking.

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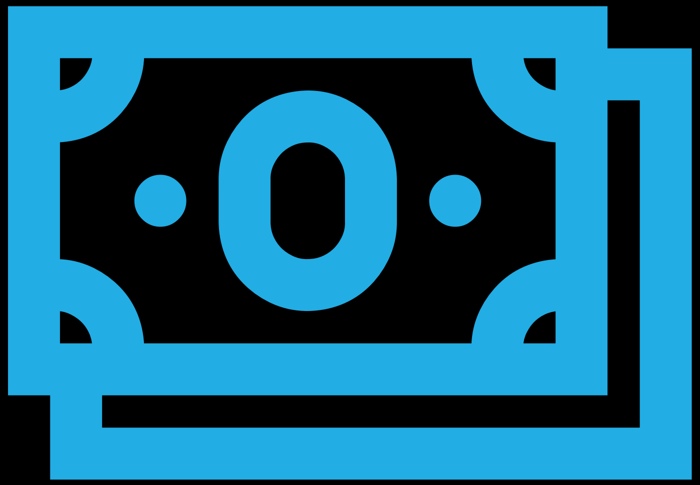
The Time Value of Money



The Time Value of Money

- Introduction
- FV of a Single Cash Flow
- FV of a Series of Cash Flows
- PV of a Single CF
- PV of a Series of CFs
- Rates, Periods, and Size of Annuity Payments

Introduction

- In short, the calculation of the time value of money involves finding equivalence between cash flows occurring on different dates.
- The real risk-free rate reflects the time preferences of individuals for current versus future real consumption.

$$
\begin{aligned}
&\textit{interest rate} \\
&= \textit{real rate} + \textit{inflation premium} + \textit{default risk premium} \\
&+ \textit{liquidity premium} + \textit{maturity premium}
\end{aligned}
$$

FV of a Single Cash Flow

- A single CF or lump-sum investment
- Principle
- Interest
- (Frequency of) Compounding

$$FV_N = PV\left(1 + \frac{r_a}{m}\right)^{mN}$$

Effective Annual Rate (EAR)

- A stated annual interest rate will result in different Effective Annual Rates (EAR) depending on the compounding frequency.

$$EAR_{DT} = \left(1 + \frac{r_a}{m}\right)^m - 1$$

- For continuous-time case:

$$EAR_{CT} = e^{r_a} - 1 > EAR_{DT}$$

FV of a Series of Cash Flows

- Annuity and Perpetuity.
- Equal CFs case:

$$FV_N = A\sum_{t=0}^{N-1}[(1+r)^t]$$

$$FV_N = A[\frac{(1+r)^N - 1}{r}]$$

FV of a Series of Cash Flows

- Unequal CFs case:

$$FV_N = \sum_{t=1}^{T} CF_t (1 + r)^{T-t}$$

From FV to PV

- Do the opposite.

$$PV_N = A\sum_{t=1}^{N}[\frac{1}{(1+r)^t}]$$

$$PV_N = A[\frac{1-\frac{1}{(1+r)^N}}{r}]$$

From FV to PV

- Unequal CFs case:

$$PV_N = \sum_{t=1}^{T} CF_t(1+r)^{-t}$$

$$FV_N = PV(1+r)^N$$

The equation above will yield the same value as the one you calculated two slides earlier.

From FV to PV

- Infinite case (when interest rates are positive):

$$PV = A\sum_{t=1}^{\infty}[\frac{1}{(1+r)^t}]$$

$$PV = \frac{A}{r}$$

Consol Bond

- There used to be such bond issued by British government that promised to pay a level CF forever. Say the bond paid £100 per year in perpetuity, how would you price the bond if the required rate of return were 5%?

Consol Bond

- What if the first payment starts at t=5?

$$PV_4 = 2000$$

$$PV_0 = 2000/(1.05)^4$$

Application

- Now you know all the essential equations in the field of time value of money.
- By now, you should know how to use CF and r to get PV/FV.
- So automatically, you know how to use PV and FV to get r.
- If you know PV, FV, and r, you know N.
- If you know PV, r, and N, you know A.
- ...

Statistical Concepts and Market Returns

Moments

- Mean
- Dispersion A.K.A Spread
- Skewness
- Kurtosis

Data

- Population
- Sample
- Sample Statistics
- Frequency Distribution

$$R_t = \frac{P_t + D_t - P_{t-1}}{P_{t-1}}$$

Measures of Mean

- Arithmetic Mean and Geometric Mean:

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$G = \sqrt[n]{\prod_{i=1}^{n} X_i} \Rightarrow Ln\ G = \frac{\sum_{i=1}^{n} x_i}{n}$$

Measures of Mean

- Why Geometric? Consider the following: You are holding a stock that worth $100 at t=0. At t=1 it worth $200, but it drops back to $100 at t=2. What's the difference between using arithmetic and geometric?

$$AM = \frac{[1 + (-0.5)]}{2} = 0.25$$

$$GM = \left((1 + 1)(1 - 0.5)\right)^{\frac{1}{2}} - 1 = 0$$

Measures of Dispersion

- Range
- Mean Absolute Deviation
- Variance
- Standard Deviation

Measures of Dispersion

$$R = MAX - MIN$$

$$MAD = \frac{\sum |X - \bar{X}|}{n - 1}$$

$$s^2 = \frac{\sum (X - \bar{X})^2}{n - 1}$$

To measure sample variance, we need to consider the degree of freedom, to make it an unbiased estimator of population variance.

Probability Concepts

Risk, Uncertainty, and Probability

- Corporate Finance ≅ Risk Management

Probability

- Random Variable – S1(a,b,c), S2(x,y,z)
- Outcomes – a, b, c, x, y, z
- Event – specific set of outcomes A-(a,b) B-(x)

- Unconditional Probability A.K.A Marginal Probability P(A)
- Conditional Probability P(A|B)
- Joint Probability P(AB)

Probability

- Multiplication Rule for Probability:

$$P(AB) = P(A|B)P(B)$$
$$P(AB) = P(A)P(B)$$

- Addition Rule for Probabilities:

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

Expected Value

- Your portfolio:

$$P = c(w_1, w_2, \dots)$$

$$\sum w_i = 1$$

- The expected return of this portfolio:

$$E(R_p) = E(w_1 R_1 + w_2 R_2 + \cdots) = w_1 E(R_1) + w_2 E(R_2) + \cdots$$

Covariance

- Definition:

$$Cov(R_i, R_j) = E\left[(R_i - ER_i)\left(R_j - ER_j\right)\right] = \sigma_{ij}$$

- Recall, for sample:

$$Cov(R_i, R_j) = \sum_{i=1}^{N} (R_i - \bar{R}_i)(R_j - \bar{R}_j)/(n-1)$$

Variance of Portfolio

- In general:

$$\sigma^2(R_P) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j Cov(R_i, R_j)$$

- The simplest case (two assets):

$$\sigma^2(R_P) = w_1^2\sigma^2(R_1) + w_2^2\sigma^2(R_2) + 2w_1w_2Cov(R_1, R_2)$$

Correlation

$$\rho(R_1, R_2) = \frac{Cov(R_1, R_2)}{\sigma(R_1)\sigma(R_2)} \in [-1,1]$$

*Bayes' Formula

- Recall:

$$P(AB) = P(A|B)P(B)$$

- Financial intuition:

$$P(EVENT|INFO) = \frac{P(INFO|EVENT)}{P(INFO)} P(EVENT)$$

Update prior probability of an event when receiving new information.

*Combination and Permutation

- Combination, pick r out of n:

$$C_n^r = \frac{n!}{(n-r)!\,r!} = \frac{n \cdot n-1 \cdot \ldots \cdot n-r+1}{r \cdot r-1 \cdot \ldots \cdot 1} = C_n^{n-r}$$

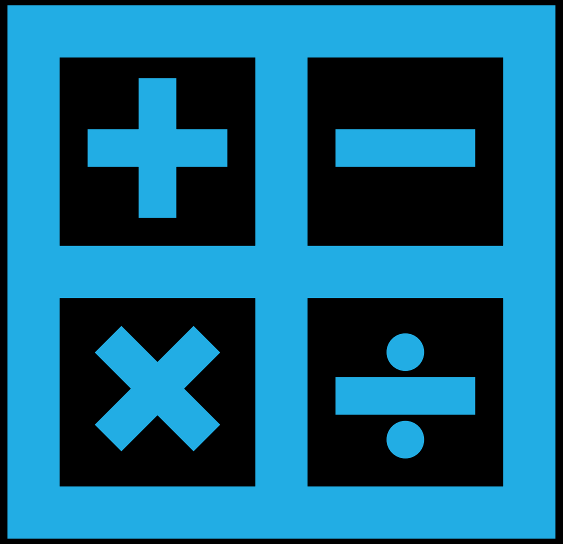
$$C_5^2 = \frac{5 \cdot 4}{2 \cdot 1} = 10 = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = C_5^3$$

*Combination and Permutation

- Permutation, pick r out of n:

$$P_n^r = \frac{n!}{(n-r)!} = n \cdot n - 1 \cdot \ldots \cdot n - r + 1$$

$$P_5^2 = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 5 \cdot 4 = 10$$



Mathematically, this is all you need for the course.