Lecture Notes on Structural Estimation: The BPP Model, GMM, and Direct Moment Approaches

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1 Introduction

Structural estimation aims to recover the deep (or structural) parameters of an economic model directly from data using theoretical restrictions. In these notes, we focus on the framework developed by Blundell, Pistaferri, and Preston (BPP, 2008) for measuring how income shocks affect consumption and the extent to which households can partially insure against these shocks. We begin with a presentation of the full model, describe the estimation methods (GMM and direct moment calculation), compare the two approaches, and finally discuss extensions to more complicated settings which may require methods like the Simulated Method of Moments (SMM).

2 The Model

2.1 Household Optimization Problem

Consider a household i that maximizes expected lifetime utility over consumption:

$$\max_{\{C_{it}, A_{it+1}\}_{t=0}^{T}} E_0 \left[\sum_{t=0}^{T} \beta^t U(C_{it}) \right], \tag{1}$$

subject to the intertemporal budget constraint:

$$A_{it+1} = (1+r)(A_{it} + Y_{it} - C_{it}), \quad A_{i,T+1} = 0, \tag{2}$$

where

- C_{it} denotes consumption in period t,

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- Y_{it} denotes disposable income,
- A_{it} denotes assets,
- β is the discount factor, and
- r is the exogenous interest rate.

2.2 Income Process

The BPP model decomposes log real income into a predictable component and an unpredictable component:

$$\log Y_{it} = Z_{it}'\omega_t + P_{it} + v_{it}, \tag{3}$$

where:

- $Z_{it}'\omega_t$ captures predictable influences (demographics, education, time trends);
- P_{it} is the permanent (persistent) income component, and
- v_{it} is the transitory (temporary) component.

The dynamics of the permanent component are modeled as a random walk:

$$P_{it} = P_{it-1} + z_{it}, \tag{4}$$

with z_{it} having variance σ_z^2. The transitory component is often modeled using an MA(1) process:

$$v_{it} = e_{it} + \theta e_{i,t-1}, \tag{5}$$

with e_{it} having variance σ_e^2.

2.3 Consumption Process and Partial Insurance

After removing the predictable part (e.g., by regressing $\log C_{it}$ on Z_{it}), the unexplained change in log consumption is represented as:

$$\Delta c_{it} = \phi\, z_{it} + \psi\, e_{it} + \xi_{it}, \tag{6}$$

where:

- ϕ is the transmission (pass-through) parameter for permanent income shocks,
- ψ is the transmission parameter for transitory shocks,
- ξ_{it} captures *taste heterogeneity*, that is, idiosyncratic (unobserved) variation in consumption behavior.

The degree of partial insurance is reflected in the values of ϕ and ψ: for example, $\phi = 0$ implies full insurance against permanent shocks, while $\phi = 1$ indicates no insurance.

2.4 Taste Heterogeneity

The unexplained variation in consumption growth not attributable to income shocks is captured by ξ_{it}. Specifically, the variance of Δc_{it} can be decomposed as:

$$\mathrm{Var}(\Delta c_{it}) = \phi^2 \sigma_z^2 + \psi^2 \sigma_e^2 + \mathrm{Var}(\xi_{it}). \tag{7}$$

Thus, the variance of taste heterogeneity is given by:

$$\sigma_\xi^2 = \mathrm{Var}(\Delta c_{it}) - \phi^2 \sigma_z^2 - \psi^2 \sigma_e^2,$$

3 GMM Estimation of the Structural Model

3.1 The General Idea of GMM

The Generalized Method of Moments (GMM) relies on matching model-implied (theoretical) moment conditions with their sample counterparts. Suppose that for each observation i the model implies:

$$E[m_i(\theta)] = 0,$$

where $\theta = (\phi, \psi)$ and $m_i(\theta)$ is a vector of moment functions. The sample moments are:

$$g_N(\theta) = \frac{1}{N} \sum_{i=1}^{N} m_i(\theta).$$

Then, the GMM estimator chooses $\hat{\theta}$ to minimize the quadratic form:

$$J(\theta) = g_N(\theta)^\top W \, g_N(\theta),$$

where W is a weighting matrix.

3.2 Moment Conditions in the BPP Framework

In the BPP model we can derive the following moment conditions:

1. Using an instrument for the permanent income shock:

   $$S_{it} = \Delta y_{i,t-1} + \Delta y_{it} + \Delta y_{i,t+1},$$

   the model implies:

   $$E\Big[\Delta c_{it} \, S_{it}\Big] = \phi \, \sigma_z^2.$$

2. Similarly, for the transitory component:

   $$E\Big[\Delta c_{it} \, \Delta y_{i,t+1}\Big] = \psi \, \sigma_e^2.$$

The corresponding sample moment vector is:

$$g_N(\theta) = \frac{1}{N} \sum_{i=1}^{N} \begin{pmatrix} \Delta c_{it} S_{it} - \phi \, \Delta y_{it} S_{it} \\ \Delta c_{it} \Delta y_{i,t+1} - \psi \, \Delta y_{it} \Delta y_{i,t+1} \end{pmatrix}.$$

3.3 Two-Step GMM Estimation

Step 1: Identity Weighting Matrix

A simple first step is to set:

$$W = I,$$

so that

$$J(\theta) = g_N(\theta)^\top I \, g_N(\theta) = \|g_N(\theta)\|^2.$$

Minimizing $J(\theta)$ yields an initial estimator $\hat{\theta}_1$.

Step 2: Optimal Weighting

After obtaining $\hat{\theta}_1$, we estimate the asymptotic covariance matrix of the sample moments,

$$\Omega = \mathrm{Var}\left(\sqrt{N}\, g_N(\theta_0)\right).$$

The optimal weighting matrix is given by:

$$W_{opt} = \Omega^{-1}.$$

By plugging W_{opt} into the GMM objective function,

$$J(\theta) = g_N(\theta)^\top \, \Omega^{-1} \, g_N(\theta),$$

we obtain an estimator that is asymptotically efficient. The intuition is that moments which are more variable (i.e., less reliable) are given less weight through the inverse covariance matrix.

4 Direct Moment Calculation Approach

For simple models like the BPP specification, the closed-form moment conditions allow us to derive:

$$\phi = \frac{E\left[\Delta c_{it}\, S_{it}\right]}{E\left[\Delta y_{it}\, S_{it}\right]},$$

where in practice we replace the expectations with sample covariances. This *direct calculation* is straightforward and yields the same estimate as GMM when one uses the identity weighting matrix and exactly as many moments as parameters.

5 Comparison of Approaches

5.1 Direct Calculation

- Advantages:
  - Computationally simple and transparent.

  - No numerical optimization is required.

- Disadvantages:

  - Limited to cases where the theoretical (closed-form) moments are available.
  - Cannot readily exploit overidentification (when more moments than parameters are available).

5.2 GMM

- Advantages:

  - Can accommodate more moment conditions than parameters (overidentification), enabling tests of the model.
  - The two-step procedure with optimal weighting (using the inverse covariance matrix) yields estimators that are asymptotically efficient.
  - Flexible and extendable to nonlinear models.

- Disadvantages:

  - Requires numerical optimization.
  - When the model is simple, it may yield similar results to the direct calculation.

6 Extensions: Why We Need GMM and SMM

6.1 Beyond Closed-Form Moments

When the model becomes more complex (e.g., introducing nonlinear transmission of income shocks as in a quadratic consumption function or incorporating higher moments such as skewness and kurtosis), analytical derivation of the moment conditions might become intractable. In this case:

- GMM provides the flexibility to incorporate many moment conditions, even if they are nonlinear functions of θ.

- If moments cannot be obtained in closed form, the **Simulated Method of Moments (SMM)** is used: the model is simulated for each candidate θ to generate simulated moments $m(\theta)$; these are then matched to the empirical moments $\hat{m}$ by minimizing:

$$Q(\theta) = [\hat{m} - m(\theta)]^\top W [\hat{m} - m(\theta)].$$

6.2 Why Optimal Weighting is Most Efficient

In GMM, if we choose the weighting matrix as the inverse of the covariance of the moment conditions, then:

- No moment is "over-emphasized" simply because it has a larger scale.
- The asymptotic variance of the GMM estimator becomes:

  $$\left[D^\top \Omega^{-1} D\right]^{-1},$$

  where D is the Jacobian of the moment functions.
- It can be shown that this choice minimizes the asymptotic variance among all possible weighting matrices.

References:

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